$$\therefore\ x=\frac{a^2+u^2-v^2}{2a},\ y=\frac{(u^2-ad)(a-d)+dv^2+af^2-aw^2}{2af}.$$

Also, $u^2+uv+v^2=a^2$,
$u^2+uw+w^2=d^2+f^2$,
$v^2+vw+w^2=(a-d)^2+f^2$.

$$\therefore\ u=\frac{ad\pm\frac13 af\sqrt3}{\sqrt{(a^2+d^2+f^2-ad\pm af\sqrt3)}},$$

$$v=\frac{a^2-ad\pm\frac13 af\sqrt3}{\sqrt{(a^2+d^2+f^2-ad\pm af\sqrt3)}},$$ The plus sign is applicable to our solution.

$$w=\frac{d^2+f^2-ad\pm\frac13 af\sqrt3}{\sqrt{(a^2+d^2+f^2-ad\pm af\sqrt3)}},$$

Let $d=\frac12 a$, and $f=\frac12 a\sqrt3$. Hence, $u=\frac13 a\sqrt3=v=w$. Thus the values of x and y are determined, and equations (7) and (8) are solved.

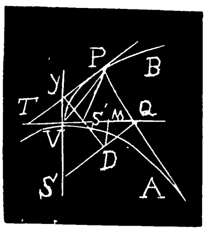
Hence $x=\dfrac{3ad^2+3af^2+3a^2d+a^2f\sqrt3+4adf\sqrt3}{6(a^2+d^2+f^2-ad+af\sqrt3)}=\frac12 a$, when $d=\frac12 a$ and $f=\frac12 a\sqrt3$.

$$y=\frac{3a^2d\sqrt3-3ad^2\sqrt3+af^2\sqrt3+3a^2f}{6(a^2+d^2+f^2-ad+af\sqrt3)}=\tfrac16 a\sqrt3,\ \text{when } d=\tfrac12 a,\ f=\tfrac12 a\sqrt3.$$

Also solved by C. N. Schmall.

323. Proposed by W. J. GREENSTREET, Marling School, Stroud, England.

S, S' are the foci of two co-vertical parabolas A and B, the axes of which are at right angles. Draw the circle K on SS' as diameter. K is cut in D and E by a straight line parallel to the axis of A such that S' lies midway between it and that axis. Show that the lines $S'D$, $S'E$ are parallel to the two tangents to A which are normals to B.



II. Solution by (1) PROPOSER, and (2) R. F. DAVIS, M. A.

I°. The axis of B is the tangent at the common vertex V to A. Hence if PQ be normal at P to B, meeting the axis of B in Q, and if PQ is at the same time a tangent to A, it necessarily follows that SQ is perpendicular to PQ and therefore parallel to TP (the tangent at P to B). Or,

II°. $S'T=S'P=S'Q$ (letters as above). Draw $YS'D$ perpendicular to the parallels TP, SQ, and DM perpendicular to the axis of B (Y, of course, being on the tangent at V to B). Then $S'M=VS'$, and D is on the circle, SS' diameter, since $\angle SDS'=90°$.

III. Solution by the PROPOSER.

The foci S, S' of (A), $y^2=4ax$, and (B) $x^2=4by$, are, respectively, $(a, 0)$, $(0, b)$, and the circle in question has as its equation $x^2+y^2-ax-by=0$.

Since the common vertex V is $(0, 0)$ and S' is $(0, b)$, the point M in which DE cuts the axis of B is $0, 2b$, and the line DE is $y=2b$.

This cuts the circle where $x^2-ax+2b^2=0$...(1), giving D and E. If the roots of this be x_1 and x_2, D, E are $(x_1, 2b)$, $(x_2, 2b)$, and $S'D$, $S'E$ have respectively, for their equations,

$$bx-x_1y+x_1b=0 \text{ and } bx-x_2y+x_2b=0.$$

The tangent at $(at^2, 2at)$ to A is $x-ty+at^2=0$.

The normal at $(2bt', at'^2)$ to B is $x+t'y-2bt'-bt'^3=0$.

These are the same line if $1=\dfrac{-t}{t'}=\dfrac{-at^2}{bt'(2+t'^2)}$.

$\therefore t=-t'$, and rejecting the values $t=t'=0$ we have $t^2b-at+2b=0$...(2). The roots of (1) and (2) are the same, and the property is proved. Perhaps Prof. Zerr will reconsider the solution he offers.

CALCULUS.

263. Proposed by V. M. SPUNAR, M. S., C. E., East Pittsburg, Pa.

Find a point such that the sum of the squares of its distances from n given points shall be a minimum, and prove that the value so found is $1/n$th part of the sum of the squares of the mutual distances between the n points, taken two and two.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

This problem is proposed and solved in Williamson's *Differential Calculus*, 9th edition, page 192, Art. 157.

Taking one of the points as origin, and the axes rectangular, let (x, y) be the coordinates of the required point. Let (a_1, b_1); (a_2, b_2); (a_3, b_3); ..., (a_{n-1}, b_{n-1}) be the coordinates of the other $(n-1)$ points. Then

$$x^2+y^2+(x-a_1)^2+(y-b_1)^2+(x-a_2)^2+(y-b_2)^2+\dots$$
$$+(x-a_{n-1})^2+(y-b_{n-1})^2=u=\text{minimum, or}$$

$$nx^2+ny^2-2(a_1+a_2+\dots+a_{n-1})x-2(b_1+b_2+\dots+b_{n-1})y$$
$$+a_1^2+a_2^2+\dots+a^2_{n-1}+b_1^2+b_2^2+\dots+b^2_{n-1}=u=\text{minimum.}$$

$$\therefore (nx-a_1-a_2-\dots-a_{n-1})dx+(ny-b_1-b_2-\dots-b_{n-1})dy=0.$$

$$\therefore x=\frac{a_1+a_2+\dots+a_{n-1}}{n},\ y=\frac{b_1+b_2+\dots+b_{n-1}}{n}.$$